M.Sc./Ph.D. Projects and Project Areas

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1. **Reversibility**
   In a given group of maps, which ones are conjugate to their own inverses? This is a rich source of interesting questions. See the document at http://www.maths.nuim.ie/staff/aof/reversibility_08.pdf about this.

2. **Harmonic Approximation**
   Given a compact set $K \subset \mathbb{R}^d$ and a continuous function $f : K \to \mathbb{R}$, the problem is to decide when f is the uniform limit on K of a sequence of functions $\{f_n\}$, each harmonic near K.

   Steps:
   1. Learn some potential theory, including capacity theory.
   2. Learn about Brownian motion.
   3. Understand known solutions to this problem in terms of Brownian motion.
   4. Understand why this is not enough.
   5. Think about the open problem of a solution in terms of capacities.

3. **The f^2 problem**
   Suppose $K \subset \mathbb{C}$ is compact, and let $R(K)$ denote the algebra of all uniform limits on K of sequences of rational functions with poles off K. Suppose f is continuous on K and $f^2 \in R(K)$. Must $f \in R(K)$?

This relates to generalisation of Rado's theorem, and there are some partial results.
For an M.Sc., understand and explain the history. For a Ph.D., make some serious progress.

4. **The graph of a direction-reversing homeomorphism of $\mathbb{C}$**
   Let $f : \mathbb{C} \to \mathbb{C}$ be a homeomorphism of degree -1. Must the graph of f be polynomially–convex? There are results for smooth f. There are many other problems about polynomial convexity.

5. **Negative Lipschitz Spaces**
   The ordinary Lipα spaces fit into a natural scale that extends the definition to all $\alpha \in \mathbb{R}$. Elements of Lipα for $\alpha < 0$ represent rather horrible distributions that might be useful for handling things like cracks in metal sheets.

   Understand the idea, and then see what you can do with it. One suggestion is to focus a behaviour of pseudo–differential operators near singularities on the boundary.

6. **Algebras of Smooth Functions**
   Problems of extension and approximation for algebras of highly–differentiable functions on smooth manifolds, and related geometric problems. Applications of the higher–order tangent concept introduced by O'Farrell–Watson to singularities of analytic sets.